Table 3. Example of multiset decision table.

Table 17: Rules generated for UPPER(D₁).

r₁: (b=0) and (c=1) → d= 1

1. Use quality of upper approximation
2. Use probability:
   e.g.: for rule 1, we have 5/8 * 8/28 = 5/28
   for rule 2: 5/10 * 10/28 = 5/28
3. Use entropy function:
   e.g.: for rule 1:  -[5/8 log 5/8 + 2/8 log 2/8 + 1/8 log 1/8] = 1.30 bits
Bayes’ Theorem:

\[ P(H \mid E) = \frac{P(H, E)}{P(E)} \]

\[ = \frac{P(E \mid H) P(H)}{P(E)} \]

Rough Membership Function [Pawlak & Skowron, 1994]

Let \( S = (U, A) \), \( B \subseteq A \) and \( X \subseteq U \)

Then the rough membership function \( \mu^B_X \) for \( X \) is a mapping from \( U \) to \([0, 1]\),

\[ \mu^B_X : U \rightarrow [0, 1] \]

For all \( e \) in \( U \), the degree of \( e \) belongs to \( X \) in light of the set of attributes \( B \) is defined as

\[ \mu^B_X(e) = \frac{|B(e) \cap X|}{|B(e)|} \]

where \( B(e) \) denotes the block containing \( e \).

Properties of rough membership function:

1. \( \mu^B_X(e) = 1 \) iff \( e \) in \( B \cdot (X) \)
2. \( \mu^B_X(e) = 0 \) iff \( e \) in \( U - B^*(X) \)
3. \( 0 < \mu^B_X(e) < 1 \) iff \( e \) in \( BN_B(X) \)
4. \( \mu^B_{U \cdot X}(e) = 1 - \mu^B_X(e) \) for any \( e \) in \( U \)
5. \( \mu^B_{X \cup Y}(e) \geq \max(\mu^B_X(e), \mu^B_Y(e)) \) for any \( e \) in \( U \)
6. \( \mu^B_{X \cap Y}(e) \leq \min(\mu^B_X(e), \mu^B_Y(e)) \) for any \( e \) in \( U \)

where \( B \cdot (X) \) is the lower approximation of \( X \) in \( B \) and \( B^*(X) \) is the upper approximation of \( X \) in \( B \).

Let \( C \rightarrow D \) be a decision rule and let \( e \) in \( U \).

Support of \( C \rightarrow_e D \) is defined as
\( \text{supp}_e(C, D) = |C(e) \cap D(e)| \)

e.g.:
support of the following rule is 3,

\( r1: (b=0) \text{ and } (e=0) \text{ and } (f=1) \rightarrow d = 1 \)

**Strength** of \( C \rightarrow_e D \):

\[
\sigma_e(C, D) = \frac{|C(e) \cap D(e)|}{|U|} = \frac{\text{supp}_e(C, D)}{|U|}
\]

e.g.: strength of \( r1 \) is 3/28

**Certainty** of \( C \rightarrow_e D \):

\[
\text{cer}_e(C, D) = \frac{|C(e) \cap D(e)|}{|C(e)|} = \mu_{C \rightarrow D(e)}(e)
\]

e.g.: certainty of \( r1 \) is 3/6

**Inverse decision rule:**

If \( C \rightarrow_e D \) be a decision rule, then \( D \rightarrow_e C \) is an inverse decision rule of \( C \rightarrow_e D \).

**Coverage** of \( C \rightarrow_e D \):

\[
\text{cov}_e(C, D) = \frac{|D(e) \cap C(e)|}{|D(e)|} = \mu_{D \rightarrow C(e)}(e)
\]

e.g.: coverage of \( r1 \) is 6/16 = 3/8

**Properties of Decision Rules:**

Let \( C \rightarrow_e D \) be a decision rule, then

1. \( \sum_{y \in C(e)} \text{cer}_y(C, D) = 1 \)
// w1 + w2 + w3 = 1

2. \[ \sum_{y \in D(e)} \text{cov}_y(C, D) = 1 \]

3. \[ \pi(D(e)) = \sum_{y \in C(e)} \text{cer}_y(C, D) \cdot p(C(y)) = \sum_{y \in C(e)} \sigma_y(C, D) \]

4. \[ \pi(C(e)) = \sum_{y \in D(e)} \text{cov}_y(C, D) \cdot p(D(y)) = \sum_{y \in D(e)} \sigma_y(C, D) \]

5. \[ \text{cer}_e(C, D) = \text{cov}_e(C, D) \cdot \pi(D(e)) / \sum_{y \in D(e)} \text{cov}_y(C, D) \cdot p(D(y)) \]

\[ = \sigma_e(C, D) / \sum_{y \in D(e)} \sigma_y(C, D) \]

\[ = \sigma_e(C, D) / \pi(C(e)) \]

6. \[ \text{cov}_e(C, D) = \text{cer}_e(C, D) \cdot \pi(C(e)) / \sum_{y \in C(e)} \text{cer}_y(C, D) \cdot p(C(y)) \]

\[ = \sigma_e(C, D) / \sum_{y \in C(e)} \sigma_y(C, D) \]

\[ = \sigma_e(C, D) / \pi(D(e)) \]

e.g.: [Pawlak, 2002]

Given the decision table

<table>
<thead>
<tr>
<th>FACT</th>
<th>Disease</th>
<th>Age</th>
<th>Sex</th>
<th>Test</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>old</td>
<td>man</td>
<td>+</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>middle</td>
<td>woman</td>
<td>+</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>old</td>
<td>man</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>old</td>
<td>man</td>
<td>-</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>no</td>
<td>young</td>
<td>woman</td>
<td>-</td>
<td>220</td>
</tr>
<tr>
<td>6</td>
<td>yes</td>
<td>middle</td>
<td>woman</td>
<td>-</td>
<td>60</td>
</tr>
</tbody>
</table>

And the following rules:

1. if (disease, yes) and (age, old) then (test, +)
2. if (disease, yes) and (age, middle) then (test, +)
3. if (desease, no) then (test, -)
4. if (disease, yes) and (age, old) then (test, -)
5. if (disease, yes) and (age, middle) then (test, -)
<table>
<thead>
<tr>
<th>Rule</th>
<th>Disease</th>
<th>Age</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>old</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>middle</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>old</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>yes</td>
<td>middle</td>
<td>-</td>
</tr>
</tbody>
</table>

We have the following certainty and coverage factors:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Strength</th>
<th>Certainty</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.44</td>
<td>0.92</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.56</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>1</td>
<td>0.76</td>
</tr>
<tr>
<td>4</td>
<td>0.04</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>0.07</td>
<td>0.44</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Flow Graph of decision rules:

<table>
<thead>
<tr>
<th>LHS</th>
<th>Strength</th>
<th>Certainty</th>
<th>Coverage</th>
<th>RHS(Facts)</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1-1</td>
<td>0.44</td>
<td>0.92</td>
<td>0.83</td>
<td>1,2</td>
<td>+</td>
</tr>
<tr>
<td>N1-4</td>
<td>0.04</td>
<td>0.08</td>
<td>0.1</td>
<td>3,4,5,6</td>
<td>-</td>
</tr>
<tr>
<td>N2-2</td>
<td>0.09</td>
<td>0.56</td>
<td>0.17</td>
<td>1,2</td>
<td>+</td>
</tr>
<tr>
<td>N2-5</td>
<td>0.07</td>
<td>0.44</td>
<td>0.14</td>
<td>3,4,5,6</td>
<td>-</td>
</tr>
<tr>
<td>N3-3</td>
<td>0.35</td>
<td>1</td>
<td>0.76</td>
<td>3,4,5,6</td>
<td>-</td>
</tr>
</tbody>
</table>

References